Test 2 Review Sheet Answers

Arithmetic Models:

1. Subtraction - Number Line: You are given a second half figure and asked to find the first.
2. Multiplication - Cartesian Product: You are finding combinations between two different sets.
3. Multiplication - Cartesian Product: You are finding combinations between two different sets.
4. Multiplication - Array: You are finding the number of elements in a figure arranged in rows and columns.
5. Subtraction - Take Away: The $4.00 was removed from the total bill. Missing Addends is also a fairly reasonable answer, by considering it as how much is left to pay.
6. Subtraction - Take Away: You removed 4 apples from the set of 9.
7. Addition - Set Model: You are combining the elements in two disjoint sets.
8. Addition - Set Model: You are combining the elements in two disjoint sets.
9. Addition - Number Line: You added 3 floors onto the previous 4 floors.
10. Multiplication - Array: You are finding the number of elements in a figure arranged in rows and columns.
11. Multiplication - Repeated Addition: You are doing $21 + 21 + 21$, but this is not a situation where you would be adding 21 onto a previous 21. So, it is more discrete.
12. Subtraction - Take Away: You removed 3 letters from the set of 50.
13. Division - Partition: You are given the number of groups and are finding how much can go in each group.
14. Subtraction - Take Away: The 2 pounds were removed from the 15.
15. Addition - Set Model: You are combining the elements in two disjoint sets.
16. Addition - Number Line: You are adding 5 more strawberries to your current 6.
17. Multiplication - Repeated Addition: You are doing $3+3+3+3$. However, if you consider stacking the magnets one by one on a scale, this could be considered more of a number line problem. The weights would be added onto the previous weights in a continuous manner.
18. Subtraction - Number Line: You are given a second half figure and asked to find the first.
19. Subtraction - Missing Addends: You are finding out how much more is needed to get to a value.
20. Subtraction - Number Line: You are given a second half figure and asked to find the first.
21. Addition - Number Line: You are adding 24 more cookies to the already having 16.
22. Subtraction - Missing Addends: You are finding out how much is left to finish the trip.
23. Multiplication - Repeated Addition: You are doing $3 + 3 + 3 + 3 + 3$, but this is not a situation where you would be adding 3 onto a previous 3. So, it is more discrete.
24. Multiplication - Area: This is clear by the word "area" appearing.
25. Division - Measurement: You are given the amount in each group and determining how many groups can be made.
26. Multiplication - Area: This is clear by the word "area" appearing.
27. Multiplication - Cartesian Product: You are finding combinations between two different sets.
28. Addition - Number Line: You are adding the 2 pounds onto the original 8 pounds.
29. Subtraction - Missing Addends: You are finding out how much is left to eat them all.
30. Multiplication - Number Line: You are considering a continuous stream of water coming out for 30 seconds adding onto how much came out before.
31. Subtraction - Missing Addends: You are finding out how much is left in order to afford the car.
32. Subtraction - Comparison: You are comparing the amounts in two different sets.
33. Addition - Set Model: You are combining the elements in two disjoint sets.
34. Subtraction - Comparison: You are comparing the amounts in two different sets.
35. Addition - Set Model: You are combining the elements in two disjoint sets.

36. Multiplication - Array: You are finding the number of elements in a figure arranged in rows and columns.
37. Multiplication - Number Line: You are considering a continuous movement for 3 hours adding onto the movement from before.
38. Addition - Number Line: You are adding the 38 pages onto the original 51 pages.
39. Subtraction - Comparison: You are comparing the amounts in two different sets.
40. Division - Measurement: You are given the amount in each group and determining how many groups can be made.

<u>Set Properties:</u>

1. Closure: All sums are in the set.
   Commutative: Order of addition can be reversed.
   Associative: Adding first two or second two in a sum of three numbers gives the same value.
   Identity: 0 is in the set, and $0 + a = a + 0 = a$.
2. Closure: All sums are in the set.
   Commutative: Order of addition can be reversed.
   Associative: Adding first two or second two in a sum of three numbers gives the same value.
   No Identity: 0 is not in the set, and no other number can act in its place.
3. No Closure: $0 - 1 = {}^{-}1$, which is not in the set.
   Not Commutative: $1 - 0 \neq 0 - 1$
   Not Associative: $(1 - 1) - 1 = 0 - 1 = {}^{-}1$, but $1 - (1 - 1) = 1 - 0 = 1$.
   No Identity: 0 is the only possibility, and $1 - 0 \neq 0 - 1$.
4. Closure: All products are in the set.
   Commutative: Order of multiplication can be reversed.
   Associative: Multiplying first two or second two in a product of three numbers gives the same value.
   Identity: 1 is in the set, and $1 \times a = a \times 1 = a$.
5. Closure: All products are in the set. (Notice that $a^2b^2 = (ab)^2$.)
   Commutative: Order of multiplication can be reversed.
   Associative: Multiplying first two or second two in a product of three numbers gives the same value.
   Identity: 1 is in the set, and $1 \times a = a \times 1 = a$.
6. No Closure: $1 \div 2 = \frac{1}{2}$, which is not in the set.
   Not Commutative: $1 \div 2 \neq 2 \div 1$
   Not Associative: $(2 \div 2) \div 2 = 1 \div 2 = \frac{1}{2}$, but $2 \div (2 \div 2) = 2 \div 1 = 2$.
   Identity: 1 is the only possibility, and $1 \div 2 \neq 2 \div 1$.
7. No Closure: $2 + 5 = 7$, which is not in the set.
   Commutative: Order of addition can be reversed.
   Adding first two or second two in a sum of three numbers gives the same value.
   Identity: 0 is in the set, and $0 + a = a + 0 = a$.
8. No Closure: $2 \times 6 = 12$, which is not in the set.
   Commutative: Order of multiplication can be reversed.
   Multiplying first two or second two in a product of three numbers gives the same value.
   No Identity: 1 is not in the set, and no other number can act in its place.

Base 5 Addition

1. 401_{five}
2. 1200_{five}
3. 4443_{five}
4. 3021_{five}
5. 4302_{five}
6. 4402_{five}

Base 5 Subtraction

1. 30_{five}
2. 102_{five}
3. 1103_{five}
4. 2343_{five}
5. 1001_{five}
6. 404_{five}

Base 5 Multiplication

1. 422_{five}
2. 3413_{five}
3. 12221_{five}
4. 20240_{five}
5. 324021_{five}
6. 11421012_{five}

Base 5 Division

1. 14_{five}
2. 23_{five} R2_{five}
3. 310_{five} R2_{five}
4. 34_{five} R10_{five}
5. 44_{five} R2_{five}
6. 30_{five} R121_{five}

The Standard Algorithms

1. When we are in the hundreds column, we end up with 11 hundreds. So, we convert 10 hundreds to 1 thousand, leaving 1 hundred.
2. When we are in the ones column, we end up with 10 ones. So we convert these to 1 ten. Now, we have 10 tens, so we convert these to 1 hundred. We are left with 7 hundreds, 0 tens, and 0 ones.
3. When we are in the tens column, we take one of the 2 hundreds and make it into 10 tens, leaving 1 hundred. We then take the 8 tens from the 10 tens, leaving 2 tens.
4. When we are in the tens column, we can't take from the hundreds column yet. So, we take one of the 4 thousands and make it 10 hundreds, then take one of the 10 hundreds and make it 10 tens. This leaves us with 3 thousands, 9 hundreds, and 12 tens, and we can then finish the subtraction.
5. Consider $1885 + 1885 + 1885 + 1885$. In the ones column, $5 + 5 + 5 + 5 = 20$, so we put 0 units and regroup to make 2 tens. This is the same as doing $5 \times 4 = 20$ and putting the 2 above the rightmost 8. In the tens column, $8 + 8 + 8 + 8 = 32$, and we also add the 2 extra tens, giving us 34. So, we keep 4 tens and regroup 30 of them into 3 hundreds. This is the same as doing $8 \times 4 = 32$ then adding the 2 to get 34, then writing down the 4 and writing the 3 above the leftmost 8. In the hundreds column, $8+8+8+8 = 32$, and we also add the 3 extra hundreds, giving us 35. So, we keep the 5 hundreds and regroup 30 of them into 3 thousands. This is the same as doing $8\times4 = 32$ and then adding the 3 to get 35, then writing down the 5 and writing the 3 above the 1. In the thousands column, $1+1+1+1 = 4$, and we also add the 3 extra thousands, giving us 7. This is the same as doing $1 \times 4 =$ and then adding the 3 to get 7.
6. We first multiply 4×8 to get 32. We write the 2 below and the 3 above the 2. This is like taking 8 units by 4 units to get 32 units, then we convert 30 of them into 3 longs. Next, we multiply 2×8 to get 16, then we add the 3 to get 19. We write the 9 below and put the 1 in the hundreds place. This is like taking 2 longs by 8 units to get 16 longs, then add the 3 longs from before to get 19 total. We then convert the extra 10 longs into 1 flat. We now start a new line with a 0 to denote that we are actually multiplying 24×10 rather than 24×1. We multiply 4×1 to get 4, which is like multiplying 4 units by 1 long to get 4 longs. We then multiply 2×1 to get 2, which is like multiplying 2 longs by 1 long to get 2 flats. Finally, we add the 1 flat, 9 longs, and 2 units to the 2 flats, 4 longs, and 0 units. We get 3 flats, 13 longs, and 2 units, which we regroup into 4 flats, 3 longs, and 2 units.

The Lattice Algorithms

1. When we add $8 + 3$, a 1 goes in the thousands diagonal to denote the regrouping and the other 1 goes in the hundreds diagonal to denote that we actually just added $800 + 300$. When we add $4 + 2$, we put a 0 in the hundreds diagonal to denote that there was no regrouping and a 6 in the tens diagonal to denote that we actually just added $40 + 20$. The units place is similar.
2. This algorithm breaks 24×18 into $20 \times 10 + 20 \times 8 + 4 \times 10 + 4 \times 8$. The thousands diagonal contains a 0 from the fact that 20×10 produced 200, which has 0 thousands. The hundreds diagonal contains a 2 from 20×10, a 1 from the regrouping in 20×8, and a 0 from the fact that no regrouping took place in 10×4. The tens diagonal contains a 3 from the regrouping of 4×8, a 6 from the leftover tens in

20×8, and a 4 from 10×4. The units diagonal contains the 2 leftover units after regrouping in 4×8.

3. This algorithm breaks 152×67 into $100 \times 60 + 100 \times 7 + 50 \times 60 + 50 \times 7 + 2 \times 60 + 2 \times 7$. $100 \times 60 = 6000$, which places a 0 in the ten-thousands diagonal and a 6 in the thousands diagonal. $100 \times 7 = 700$, which places a 0 in the thousands diagonal and a 7 in the hundreds diagonal. $50 \times 60 = 3000$ (30 hundreds), which places a 3 in the thousands diagonal (regrouping of 30) and the remaining 0 in the hundreds diagonal. $50 \times 7 = 350$ (35 tens), which places a 3 in the hundreds diagonal (regrouping of 30) and the remaining 5 in the tens diagonal. $2 \times 60 = 120$ (12 tens), which places a 1 in the tens diagonal (regrouping of 10) and the remaining 3 in the tens diagonal. $2 \times 7 = 14$, which places a 1 in the tens diagonal (regrouping of 10) and the remaining 4 in the units diagonal.

<u>The Equal Additions Algorithm</u>

1. We add 20 to both numbers in order to make 82 become 102. We want a 0 in the tens column so that we no longer have to regroup. This gives us 227 - 102, which is an equivalent problem because we basically did $(207 + 20) - (82 + 20)$, which is the same as $207 + 20 - 82 - 20 = 207 - 82$. We merely added and subtracted 20 from the problem.
2. We add 50 to both numbers in order to get a 0 in the tens place. This gives us 4074-1301, so we now add 700 to both numbers in order to get a 0 in the hundreds place. This leaves us with with 4774-2001, which is an equivalent problem for the same reason as above. Alternatively, we can consider these as shifts on the number line, but the distance between the numbers does not change.

<u>Long Division Algorithm</u>

1. We have 4 tens and 6 units that we are dividing into 2 groups. We first put 2 tens in each group, using up all 4 and leaving no others. We then put 3 units in each group, using up all 6 and leaving no others. There is no remainder and we have 23 in each group.
2. We have 6 hundreds, 2 tens, and 1 unit that we are dividing into 2 groups. We first put 3 hundreds in each group, using up all 6 and leaving no others. We then consider the 2 tens. We put 0 tens in each group (not enough to put 1 in each) and convert the remaining 2 into 20 units. We now have 21 units, so we put 7 units in each up, leaving no others. There is no remainder and we have 307 in each group.
3. We have 1 thousand, 2 hundreds, 6 tens, and 8 units that we are dividing into 5 groups. We can't put thousands in the group, so we convert it to 10 hundreds, giving us 12 hundreds. We then put 2 hundreds in each group, using up 10 hundreds and leaving 2 hundreds. We then convert the 2 hundreds to 20 tens, giving us 26 tens. Next, we put 5 tens in each group, using up 25 tens and leaving 1 ten. We then convert the 1 ten into 10 units, giving us 18 units total. Finally, we put 3 units in each group, using up 15 units and leaving 3 units. So, we have 253 in each group and 3 units remaining.

<u>Divisibility Tests</u>

1. (2) 540 ends in 0, so $2 \mid 540$.
   (3) $5 + 4 + 0 = 9$ and $3 \mid 9$, so $3 \mid 540$.
   (4) $4 \mid 40$, so $4 \mid 540$.
   (5) 540 ends in 0, so $5 \mid 540$.
   (6) $2 \mid 540$ and $3 \mid 540$, so $6 \mid 540$.
   (7) $54 - 2 \cdot 0 = 54$. $7 \nmid 54$, so $7 \nmid 540$.
   (8) $8 \nmid 540$ (divisibility test is no help here, so use long division).
   (9) $5 + 4 + 0 = 9$ and $9 \mid 9$, so $9 \mid 540$.
   (10) 540 ends in 0, so $10 \mid 540$.
   (11) $5 - 4 + 0 = 1$ and $11 \nmid 1$, so $11 \nmid 540$.
2. (2) 3465 ends in 5, so $2 \nmid 540$.
   (3) $3 + 4 + 6 + 5 = 18$ and $3 \mid 18$, so $3 \mid 3465$.
   (4) $2 \nmid 3465$, so $4 \nmid 3465$.
   (5) 3465 ends in 5, so $5 \mid 3465$.
   (6) $2 \nmid 3465$, so $6 \nmid 3465$.
   (7) $346 - 2 \cdot 5 = 336$, then $33 - 2 \cdot 6 = 21$. $7 \mid 21$, so $7 \nmid 3465$.
   (8) $2 \nmid 3465$, so $8 \nmid 3465$.
   (9) $3 + 4 + 6 + 5 = 18$ and $3 \mid 18$, so $3 \mid 3465$.
   (10) 3465 ends in 5, so $10 \nmid 3465$.
   (11) $3 - 4 + 6 - 5 = 0$ and $11 \mid 0$, so $11 \mid 3465$.

3. (2) 1848 ends in 8, so $2 \mid 1848$.
   (3) $1+8+4+8=21$ and $3 \mid 21$, so $3 \mid 1848$.
   (4) $4 \mid 48$, so $4 \mid 1848$.
   (5) 1848 ends in 8, so $5 \nmid 1848$.
   (6) $2 \mid 1848$ and $3 \mid 1848$, so $6 \mid 1848$.
   (7) $184 - 2 \cdot 8 = 168$, then $16 - 2 \cdot 8 = 0$. $7 \mid 0$, so $7 \mid 1848$.
   (8) $8 \mid 848$, so $8 \mid 1848$.
   (9) $1+8+4+8=21$ and $9 \nmid 21$, so $9 \nmid 1848$.
   (10) 1848 ends in 8, so $10 \nmid 1848$.
   (11) $1-8+4-8={}^{-}11$ and $11 \mid {}^{-}11$, so $11 \mid 1848$.
4. (2) 2873 ends in 3, so $2 \nmid 2873$.
   (3) $2+8+7+3=20$ and $3 \nmid 20$, so $3 \nmid 2873$.
   (4) $2 \nmid 2873$, so $4 \nmid 2873$.
   (5) 2873 ends in 3, so $5 \nmid 2873$.
   (6) $2 \nmid 2873$, so $6 \nmid 2873$.
   (7) $287-2 \cdot 3 = 281$, then $28 - 2 \cdot 1 = 26$. $7 \nmid 26$, so $7 \nmid 2873$.
   (8) $2 \nmid 18735$, so $4 \nmid 2873$.
   (9) $2+8+7+3=20$ and $9 \nmid 20$, so $9 \nmid 2873$.
   (10) 2873 ends in 3, so $5 \nmid 2873$.
   (11) $2-8+7-3={}^{-}2$ and $11 \nmid {}^{-}2$, so $11 \nmid 2873$.
5. (2) 27720 ends in 0, so $2 \mid 27720$.
   (3) $2+7+7+2+0=18$ and $3 \mid 18$, so $3 \mid 27720$.
   (4) $4 \mid 20$, so $4 \mid 27720$.
   (5) 27720 ends in 0, so $5 \mid 27720$.
   (6) $2, 3 \mid 27720$, so $6 \mid 27720$.
   (7) $2772 - 2 \cdot 0 = 2772$, then $277 - 2 \cdot 2 = 273$, then $27 - 2 \cdot 3 = 21$. $7 \mid 21$, so $7 \mid 27720$.
   (8) $8 \mid 720$, so $8 \mid 27720$.
   (9) $2+7+7+2+0=16$ and $9 \nmid 16$, so $9 \nmid 27720$.
   (10) 27720 ends in 0, so $10 \mid 27720$.
   (11) $2-7+7-2+0=0$ and $11 \mid 0$, so $11 \mid 27720$.
6. (2) 103818 ends in 8, so $2 \mid 103818$.
   (3) $1+0+3+8+1+8=21$ and $3 \mid 21$, so $3 \mid 103818$.
   (4) $4 \nmid 18$, so $4 \nmid 103818$.
   (5) 103818 ends in 8, so $5 \nmid 103818$.
   (6) $2 \mid 103818$ and $3 \mid 103818$, so $6 \mid 103818$.
   (7) $10381 - 2 \cdot 8 = 10365$, then $1036 - 2 \cdot 5 = 1026$, then $102 - 2 \cdot 6 = 90$. $7 \nmid 90$, so $7 \nmid 103818$.
   (8) $4 \nmid 103818$, so $8 \nmid 103818$.
   (9) $1+0+3+8+1+8=21$ and $9 \nmid 21$, so $9 \nmid 103818$.
   (10) 103818 ends in 8, so $10 \nmid 103818$.
   (11) $1-0+3-8+1-8={}^{-}11$ and $11 \mid {}^{-}11$, so $11 \mid 103818$.

<u>Prime or Composite Numbers</u>

1. Composite since $169 = 13 \cdot 13$.
2. Composite since $203 = 7 \cdot 29$.
3. Prime
4. Composite since $221 = 13 \cdot 17$.
5. Composite since $1547 = 7 \cdot 221$.
6. Prime

Sieve of Eratosthenes

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~	~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~	71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~	~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~	~~111~~	~~112~~	113	~~114~~	~~115~~	~~116~~	~~117~~	~~118~~	~~119~~	~~120~~
~~121~~	~~122~~	~~123~~	~~124~~	~~125~~	~~126~~	127	~~128~~	~~129~~	~~130~~	131	~~132~~	~~133~~	~~134~~	~~135~~	~~136~~	137	~~138~~	139	~~140~~
~~141~~	~~142~~	~~143~~	~~144~~	~~145~~	~~146~~	~~147~~	~~148~~	149	~~150~~	151	~~152~~	~~153~~	~~154~~	~~155~~	~~156~~	157	~~158~~	~~159~~	~~160~~
~~161~~	~~162~~	163	~~164~~	~~165~~	~~166~~	167	~~168~~	~~169~~	~~170~~	~~171~~	~~172~~	173	~~174~~	~~175~~	~~176~~	~~177~~	~~178~~	179	~~180~~
181	~~182~~	~~183~~	~~184~~	~~185~~	~~186~~	~~187~~	~~188~~	~~189~~	~~190~~	191	~~192~~	193	~~194~~	~~195~~	~~196~~	197	~~198~~	199	~~200~~
~~201~~	~~202~~	~~203~~	~~204~~	~~205~~	~~206~~	~~207~~	~~208~~	~~209~~	~~210~~	211	~~212~~	~~213~~	~~214~~	~~215~~	~~216~~	~~217~~	~~218~~	~~219~~	~~220~~
~~221~~	~~222~~	223	~~224~~	~~225~~	~~226~~	227	~~228~~	229	~~230~~	~~231~~	~~232~~	233	~~234~~	~~235~~	~~236~~	~~237~~	~~238~~	239	~~240~~
241	~~242~~	~~243~~	~~244~~	~~245~~	~~246~~	~~247~~	~~248~~	~~249~~	~~250~~	251	~~252~~	~~253~~	~~254~~	~~255~~	~~256~~	257	~~258~~	~~259~~	~~260~~
~~261~~	~~262~~	263	~~264~~	~~265~~	~~266~~	~~267~~	~~268~~	269	~~270~~	271	~~272~~	~~273~~	~~274~~	~~275~~	~~276~~	277	~~278~~	~~279~~	~~280~~
281	~~282~~	283	~~284~~	~~285~~	~~286~~	~~287~~	~~288~~	~~289~~	~~290~~	~~291~~	~~292~~	293	~~294~~	~~295~~	~~296~~	~~297~~	~~298~~	~~299~~	~~300~~
~~301~~	~~302~~	~~303~~	~~304~~	~~305~~	~~306~~	307	~~308~~	~~309~~	~~310~~	311	~~312~~	313	~~314~~	~~315~~	~~316~~	317	~~318~~	~~319~~	~~320~~
~~321~~	~~322~~	~~323~~	~~324~~	~~325~~	~~326~~	~~327~~	~~328~~	~~329~~	~~330~~	331	~~332~~	~~333~~	~~334~~	~~335~~	~~336~~	337	~~338~~	~~339~~	~~340~~
~~341~~	~~342~~	~~343~~	~~344~~	~~345~~	~~346~~	347	~~348~~	349	~~350~~	~~351~~	~~352~~	353	~~354~~	~~355~~	~~356~~	~~357~~	~~358~~	359	~~360~~
~~361~~	~~362~~	~~363~~	~~364~~	~~365~~	~~366~~	367	~~368~~	~~369~~	~~370~~	~~371~~	~~372~~	373	~~374~~	~~375~~	~~376~~	~~377~~	~~378~~	379	~~380~~
~~381~~	~~382~~	383	~~384~~	~~385~~	~~386~~	~~387~~	~~388~~	389	~~390~~	~~391~~	~~392~~	~~393~~	~~394~~	~~395~~	~~396~~	397	~~398~~	~~399~~	~~400~~

Primes under 400: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 101, 103, 107, 109, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 347, 353, 359, 367, 373, 379, 383, 389, 397

Fundamental Theorem of Arithmetic

1. $2^2 \cdot 3^3 \cdot 5$
2. $3^2 \cdot 5 \cdot 7 \cdot 11$
3. $2^3 \cdot 3 \cdot 7 \cdot 11$
4. $13^2 \cdot 17$
5. $2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
6. $2 \cdot 3 \cdot 11^3 \cdot 13$

GCD (Intersection of Sets)

1. Factors of 4: $\{1, 2, 4\}$; Factors of 6: $\{1, 2, 3, 6\}$; GCD: 2
2. Factors of 12: $\{1, 2, 3, 4, 6, 12\}$; Factors of 18: $\{1, 2, 3, 6, 9, 18\}$; GCD: 6
3. Factors of 20: $\{1, 2, 4, 5, 10, 20\}$; Factors of 32: $\{1, 2, 4, 8, 16, 32\}$; GCD: 4

GCD (Prime Factorization)

1. $4 = 2^2$; $6 = 2 \cdot 3$; GCD: 2
2. $12 = 2^2 \cdot 3$; $18 = 2 \cdot 3^2$; GCD: $2 \cdot 3 = 6$
3. $20 = 2^2 \cdot 5$; $32 = 2^5$; GCD: $2^2 = 4$
4. $98 = 2 \cdot 7^2$; $147 = 3 \cdot 7^2$; GCD: $7^2 = 49$
5. $90 = 2 \cdot 3^2 \cdot 5$; $84 = 2^2 \cdot 3 \cdot 7$; GCD: $2 \cdot 3 = 6$
6. $1800 = 2^3 \cdot 3^2 \cdot 5^2$; $60 = 2^2 \cdot 3 \cdot 5$; GCD: $2^2 \cdot 3 \cdot 5 = 60$

LCM (Intersection of Sets)

1. Multiples of 4: $\{4, 8, 12, 16, 20, 24, ...\}$; Multiples of 6: $\{6, 12, 13, 24\}$; LCM: 12
2. Multiples of 12: $\{12, 24, 36, 48, 60, ...\}$; Multiples of 18: $\{18, 36, 54, 72, ...\}$; LCM: 36
3. Multiples of 20: $\{20, 40, 60, 80, 100, 120, 140, 160, 180, 200, ...\}$;
   Multiples of 32: $\{32, 64, 96, 128, 160, 192, ...\}$; GCD: 160

LCM (Prime Factorization)

1. $4 = 2^2$; $6 = 2 \cdot 3$; LCM: $2^2 \cdot 3 = 12$
2. $12 = 2^2 \cdot 3$; $18 = 2 \cdot 3^2$; LCM: $2^2 \cdot 3^2 = 36$
3. $20 = 2^2 \cdot 5$; $32 = 2^5$; LCM: $2^5 \cdot 5 = 160$
4. $98 = 2 \cdot 7^2$; $147 = 3 \cdot 7^2$; LCM: $2 \cdot 3 \cdot 7^2 = 294$
5. $90 = 2 \cdot 3^2 \cdot 5$; $84 = 2^2 \cdot 3 \cdot 7$; LCM: $2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$
6. $1800 = 2^3 \cdot 3^2 \cdot 5^2$; $60 = 2^2 \cdot 3 \cdot 5$; LCM: $2^3 \cdot 3^2 \cdot 5^2 = 1800$